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Grid spanners with low forwarding index for energy efficient networks*

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Abstract: A routing R of a connected graph G is a collection that contains simple paths connecting every ordered pair of vertices in G. The *edge-forwarding index with respect to* R (or simply the forwarding index with respect to R) $\pi(G, R)$ of G is the maximum number of paths in R passing through any edge of G. The *forwarding index* $\pi(G)$ of G is the minimum $\pi(G, R)$ over all routings R's of G. This parameter has been studied for different graph classes [14], [1], [7], [5]. Motivated by energy efficiency, we look, for different numbers of edges, at the best spanning graphs of a square grid, namely those with a low forwarding index.

Key-words: spanning subgraphs, forwarding index, energy saving, routing, grid

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Graphes couvrants de la grille avec un indice de transmission faible pour des réseaux efficaces en énergie

Résumé : Un routage R d'un graphe connexe G est un ensemble de chemins simples connectant toutes les paires ordonnées de sommets de G. L'*indice de transmission arête relatif à* R (ou simplement l'indice de transmission relatif à R) $\pi(G, R)$ de G est le nombre maximum de chemins de R qui utilisent une arête. L'*indice de transmission* de G, $\pi(G)$, est le $\pi(G, R)$ minimum sur tous les routages R de G. Ce paramètre a été étudié pour différentes classes de graphes [14], [1], [7], [5]. Motivés par l'efficacité énergétique des réseaux, nous étudions, pour un nombre d'aêtes donné, les meilleurs sous-graphes couvrants de la grille carré, c'est-à-dire ceux avec un indice de transmission faible.

Mots-clés : sous-graphes couvrants, indice de transmission, efficacité énergétique, routage, grille

1 Introduction

A routing R of a given connected graph G of order N is a collection of $N(N-1)$ simple paths connecting every ordered pair of vertices of G. The routing R induces on every edge e a *load* that is the number of paths going through e. The *edge-forwarding index* (or simply the forwarding index) $\pi(G, R)$ of G with respect to R is the maximum number of paths in R passing through any edge of G. It corresponds to the maximum load over all edges of the graph when R is used. Therefore, it is important to find routings minimizing this index. The forwarding index $\pi(G)$ of G is the minimum $\pi(G, R)$ over all routings R's of G. This parameter has been studied for different graph classes (examples can be found in [1], [7], [5]) and this survey [14] gives a global view on the known results.

We call a connected spanning subgraph of a graph G, a *spanner* of G. More precisely, it is a connected subgraph that has the same set of vertices as G. Our goal is to find, for a given bound on the number of edges, the best spanner of G, namely the one with the minimum forwarding index. The problem can also be viewed as: for a given bound U on the forwarding index, find a spanner F of G with minimum number of edges such that $\pi(F) \leq U$.

Knowing how to solve this problem is very interesting in practice for network operators willing to reduce the energy consumed by their networks. In fact, most of the network links consume a constant energy independently of the amount of traffic they are flowing [2], [13]. Therefore, it was proposed to reduce the energy used by the network links by turning some of them off, or more conveniently, putting them into an idle mode. Outside the rush hours, several studies [3], [4], [12], [9], [10], [11], show that a good choice of the links to turn off can lead to significant energy savings, while keeping the same communication quality. In the case where the throughputs from every node to every other node are of the same order, and where the capacities also lie in the same small range, a good choice of those links is reduced to the problem of finding spanners of the network with low forwarding indices.

In this paper, we consider the case in which the initial graph is a square grid. We consider the asymptotic case with n large. We have two main contributions.

On one side, we know that the forwarding index of the $n \times n$ grid G_n is $\frac{n^3}{2}$, see Proposition 1 [8]. G_n has $2(n-1)^2 \sim 2n^2$ edges. An important remark is that the load of the edges is lower in the corner than in the middle of the grid. Using that, we show that we can build spanners of G_n with much fewer edges (only $13/18 \approx 72\%$ of the edges) and the same forwarding indices as G_n. We show that the proposed spanners are close to optimum in the sense that we prove that it is impossible to build spanners with fewer than $4/3n^2$ edges (66% of the edges).

On the other side, the smallest possible spanner of the $n \times n$ grid G_n is a spanning tree. The forwarding index of the best spanning tree is asymptotically $\frac{3n^4}{8}$, see Proposition 2 [8]. When we add edges and consider spanners with a larger number of edges, the load on the edges decreases, and so does the forwarding index. In this paper, we study how the forwarding index decreases, when we increase the number of edges.

The following table summarizes our results:

	Spanning tree	Spanners		Grid
forwarding index	$\frac{3}{8}n^4$	$\frac{1}{2a}n^4 (2 \leq a < n)$	$\frac{1}{2}n^3$	$\frac{1}{2}n^3$
lower bound on the number of edges	$n^2 - 1$	$\simeq n^2 + \frac{4}{9}(0.1a)^2$	$\frac{12}{9}n^2$	
number of edges in the constructions	$n^2 - 1$	$n^2 + \frac{4}{9}a^2$	$\frac{13}{9}n^2$	$2n^2$

Proposition 1 *[8] The forwarding index of G_n is asymptotically $\frac{n^3}{2}$.*

Proposition 2 *[8] For $n \geq 3$, The spanning tree of G_n with the minimum forwarding index is a tree with centroid of degree 4 and 4 branches of almost equal sizes. its forwarding index is asymptotically $\frac{3n^4}{8}$.*

2 Spanners with the forwarding index of the grid, $\frac{n^3}{2}$, but much fewer edges

In this section, we first show that a spanner with the forwarding index of the grid has at least $\frac{4n^2}{3} = \frac{12n^2}{9}$ edges. We then provide spanners with $\frac{13n^2}{9}$ edges. But, before, we present some notations that will be used throughout the paper.

Notations We note by $G_n = (V_n, E_n)$ the $n \times n$ square grid, where V_n is the set of Vertices and E_n is the set of edges. A square grid can always be seen as n rows intersecting n columns. We name $v(r, c)$ the vertex at the intersection of row $r \in [n]$ with column $c \in [n]$, where $[n]$ denotes the interval of the integer numbers between 1 and n. An edge joining $v(r, c)$ to $v(r, c+1)$ is named $e_h(r, c)$ and an edge joining $v(r, c)$ to $v(r+1, c)$ is named $e_v(r, c)$.

Proposition 3 *For any F spanner of G_n such that $\pi(F) \leq \frac{n^3}{2}$, F must have, asymptotically, at least $\frac{4n^2}{3}$ edges.*

Proof: Consider F a spanner of G_n and let R be a routing of F such that $\pi(F, R) \leq \frac{n^3}{2}$. For an integer $l \in [n]$, we call *load on line l*, the sum of the load on the edges $e_v(l, j) \in E(F)$, for $j \in [n]$. The load on line l is $2l(n-l)n^2$ as there are ln vertices over line l and $(n-l)n$ vertices below. If F has $n - x_l$ edges on line l, there exists at least one of these edges with load at least $\frac{2l(n-l)n^2}{n-x_l}$. As $\pi(F, R) \leq \frac{n^3}{2}$, we should have

$$\frac{2l(n-l)n^2}{n-x_l} \leq \frac{n^3}{2}.$$

That is

$$n - x_l \geq \frac{4l(n-l)}{n}.$$

Thus, F should have at least $\sum_{l=1}^{n} \frac{4l(n-l)}{n}$ vertical edges. The same argument independently holds for the horizontal edges. Hence, a spanner of the grid, with load lower than $\frac{n^3}{2}$ on all edges, has at least

$$2\sum_{l=1}^{n} \frac{4l(n-l)}{n} \approx \frac{4n^2}{3} \text{edges.}$$

□

Theorem 1 *There exists F_n a spanner of G_n such that $\pi(F_n) \sim \frac{n^3}{2}$ and its number of edges is asymptotically equal to $\frac{13n^2}{9}$.*

Proof: Let us first explain the intuition behind the construction of the spanner of the grid, F_n. We know from the proof of Proposition 3 the ratio of edges needed in every row or column in order to satisfy the lower bound. We cut the grid into small squares. Then, according to the position of the square, we use only the number of needed horizontal edges and vertical edges in each square according to the lower bound. It turns out that adding only few edges to ensure the connectivity is enough to get a spanner F_n with a routing R such that $\pi(F_n, R) \sim \frac{n^3}{2}$.

Construction of F_n. Let k be an integer number such that $1 \leq k \leq n$. We divide G_n into small square grids of size $k \times k$. We do so by partitioning vertices of G_n into $(\frac{n}{k})^2$ sets $S_{(i,j)}$ with $i \in [\frac{n}{k}]$ and $j \in [\frac{n}{k}]$: $S_{(i,j)} = \{v(r, c) \in V_n; i - 1 < \frac{r}{k} \leq i, j - 1 < \frac{c}{k} \leq j\}$. We call a vertex in $S_{(i,j)}$ that has a neighbour in G_n outside $S_{(i,j)}$ a border vertex.

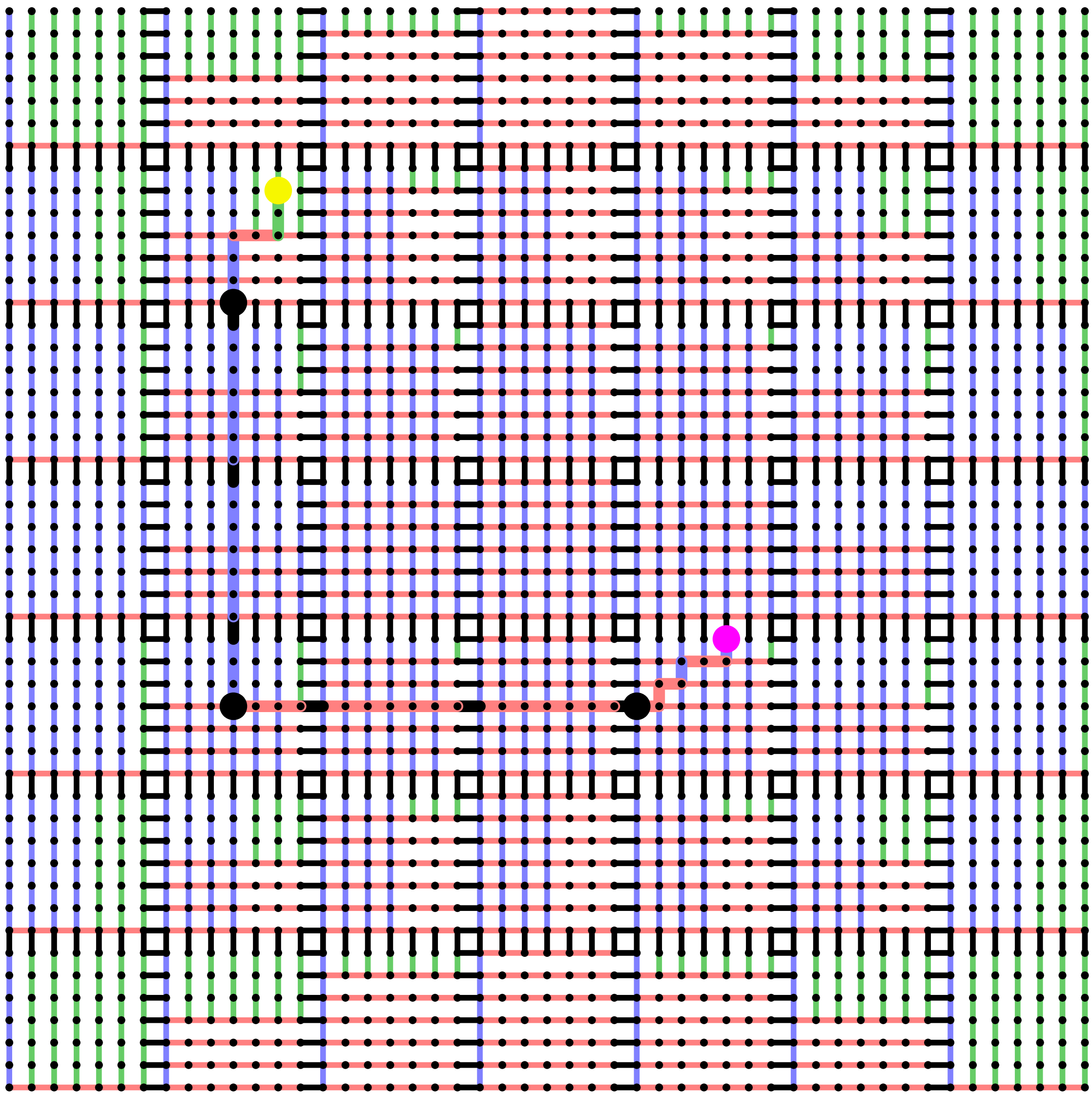

Figure 1: Construction of the spanner F_n of Theorem 1, for $n = 7^2$, and an example of path of the routing R of F_n (from the yellow vertex to the pink vertex).

Let us now describe a spanner F_n that verifies our theorem. An example of it is shown in Figure 1 in the case of $n = k^2 = 7^2$. Let t be the function defined on integers by $t(x) = \lceil 4xk(n - xk)k/n^2 \rceil$. It represents the number of needed columns (respectively rows) for a square that is on the x-th position horizontally (respectively vertically). We build F_n starting from a subgraph that has all vertices of G_n and no edges. For every $S_{(i,j)}$, $i, j \in [\frac{n}{k}]$, we choose edges to connect vertices in $S_{(i,j)}$ in the following way:

- we add to F_n all edges $e_v(r, c)$ such that $(r \mod k) \in \{1, \ldots, t(i)\}$ (red edges in Figure 1) and

- all edges $e_h(r,c)$ such that $(c \mod k) \in \{1,\ldots,t(j)\}$ (blue edges in Figure 1);

- then we add to F_n simple paths just to connect the remaining independent vertices (green edges in Figure 1).

- We then add all edges that do not have both endpoints in the same set $S_{(i,j)}$ (black edges in 1). We show in the following that adding all of them is not strictly necessary.

Description of the routing R. We now give a routing of the spanner F_n, R. For every ordered pair of vertices $(v(r_a,c_a), v(r_b,c_b))$ of V_n, we describe the path connecting $v(r_a,c_a)$ to $v(r_b,c_b)$ in R. We distinguish two types of ordered pairs of vertices:

- Type-1 pairs: $\lceil r_a/k \rceil = \lceil r_b/k \rceil$ or $\lceil c_a/k \rceil = \lceil c_b/k \rceil$. Notice that this type includes ordered pairs with vertices that belongs to the same set $S_{(i,j)}$.

- Type-2 pairs: All the ordered pairs that do not belong to the first type.

For the Type-1 pairs, R uses the shortest path routing. For Type-2 pairs, R uses a three-segment path. An example of such path is shown in Figure 1. We name $i_a = \lceil r_a/k \rceil$, $i_b = \lceil r_b/k \rceil$, $j_a = \lceil c_a/k \rceil$ and $j_b = \lceil c_b/k \rceil$:

- Step-1: Let $i_m = \min(i_a, i_b, n/k - i_a, n/k - i_b)$ and $j_m = \min(j_a, j_b, n/k - j_a, n/k - j_b)$. The first segment is the shortest path from $v(r_a,c_a)$ to one of the two border vertices of $S_{(i_a,j_a)}$ that are on row $k(i_a-1)+t(j_m)$. Among the two vertices, we choose $v(r_x,c_x)$, which has the smallest distance to $S_{(i_a,j_b)}$ (as the first black vertex on the route in Figure 1).

- Step-2: Similarly, two border vertices of $S_{(i_b,j_b)}$ are on column $k(i_b-1)+t(i_m)$. Among these two vertices, $v(r_y,c_y)$ is the one that has the smallest distance to $S_{(i_a,j_b)}$ (as the third black vertex on the route in Figure 1). The second segment will be linking $v(r_x,c_x)$ to $v(r_y,c_y)$ by using the path $[v(r_x,c_x)v(r_x,c_y)v(r_y - c_y)]$, which is the shortest path from $v(r_x,c_x)$ to $v(r_x,c_y)$ composed of the two direct paths $[v(r_x,c_x)v(r_x,c_y)]$, following row r_x, and $[v(r_x,c_y)v(r_y - c_y)]$, following column c_y.

- Step-3: The third and last segment will be the shortest path from $v(r_y,c_y)$ to $v(r_b,c_b)$.

Note that k may be an arbitrary integer between 1 and n. We choose a k such that $1 \ll k \ll \sqrt{n}$. For instance, we may choose $k = n^{1/3}$.

Number of edges of F_n. Let us compute the number of edges in the spanner, F_n. First the edges used in the subgraph induced by $S_{(i,j)}$ are all the edges on a row from 1 to $t(i)$, all edges on a column from 1 to $t(j)$, to which we add the edges that connect the rest of vertices through a spanning tree. Hence the number of edges in $S_{(i,j)}$ is:

$$
\begin{aligned}
&\approx\; t(i)\cdot k + t(j)\cdot k + (k-t(i))(k-t(j)) \\
&\approx\; k^2 + t(i)t(j) \\
&\approx\; k^2\Big(1+\frac{16ijk^2(n-ik)(n-jk)}{n^4}\Big) \\
&\approx\; k^2\Big(1+\frac{16ijk^2}{n^2}+\frac{16i^2j^2k^4}{n^4}-\frac{16i^2jk^3}{n^3}-\frac{16ij^2k^3}{n^3}\Big)
\end{aligned}
\tag{1}
$$

The sum of those edges considering all the subsets $S_{(i,j)}$ (with $i \in [\frac{n}{k}]$ and $j \in [\frac{n}{k}]$) is :

$$\begin{aligned}
&\approx k^2 \sum_{i=1}^{n/k} \sum_{j=1}^{n/k} (1 + \frac{16ijk^2}{n^2} + \frac{16i^2j^2k^4}{n^4} - \frac{16i^2jk^3}{n^3} - \frac{16ij^2k^3}{n^3}) \\
&\approx k^2[\frac{n^2}{k^2} + \frac{16k^2}{n^2}(\sum_i i)^2 + \frac{16k^4}{n^4}(\sum_i i^2)^2 - 2\cdot\frac{16k^3}{n^3}(\sum_i i^2)(\sum_i i)] \\
&\approx k^2[\frac{n^2}{k^2} + \frac{16k^2}{n^2}\cdot\frac{n^4}{4k^4} + \frac{16k^4}{n^4}\cdot\frac{n^6}{9k^6} - 2\cdot\frac{16k^3}{n^3}\cdot\frac{n^5}{6k^5}] \\
&\approx k^2[\frac{n^2}{k^2} + \frac{4n^2}{k^2} + \frac{16n^2}{9k^2} - \frac{32n^2}{6k^2}] \\
&= \frac{13}{9}n^2 + o(n^2)
\end{aligned}$$

The number of the remaining edges is $\approx\ 2\frac{n^2}{k} = o(n^2)$, as $k \gg 1$. Therefore, as stated in the theorem, the number edges of F_n is asymptotically equal to $\frac{13n^2}{9} + o(n^2)$.

Load of the edges of F_n. Lets now verify that every edge has an asymptotic load which is not greater than $\frac{n^3}{2} + o(n^3)$. Consider an edge $e_h(r,c)$ whose incident points are in $S_{(i,j)}$. The number of Type-1 pairs that may use $e_h(r,c)$ is bounded by the number of pairs having one endpoint in $S_{(i,j_a)}$ and $S_{(i,j_b)}$ for some $j_a, j_b \in [\frac{n}{k}]$ and those having one end point in $S_{(i_a,j)}$ and $S_{(i_b,j)}$ for some $i_a, i_b \in [\frac{n}{k}]$. The number of these pairs is bounded by $2k^2n^2 = o(n^3)$ (as $k^2 = o(n)$).

Then, for Type-2 pairs, we can start by the load induced by the segments of paths described previously in step-1 and step-3. This load is clearly bounded by the number of pairs having one endpoint inside $S_{(i,j)}$ and another endpoint outside $S_{(i,j)}$. The number of these pairs is bounded by: $2k^2(n-k)^2 = o(n^3)$ (as $k^2 = o(n)$). For Step-2, as the construction of the spanner F_n has the needed density of edges, the load is kept below $\frac{n^3}{2} + o(n^3)$. The same argument holds for the black edges between two adjacent subsets $S_{(i_a,j_a)}$ and $S_{(i_b,j_b)}$. This ends the proof. □

3 Spanners with forwarding indices in the range $]\frac{n^3}{2}, \frac{3n^4}{8}[$ and Lower bounds

We first provide spanners with forwarding indices in the range $]\frac{n^3}{2}, \frac{3n^4}{8}[$ in Proposition 4. We then prove that these spanners have a number of edges of the optimum order, see Proposition 5

3.1 Spanners' constructions

Proposition 4 *Let a be an integer such that, $2 \le a \le n$. There exists a spanner $F_n(a)$ of G_n with asymptotically $n^2 + \frac{4}{9}a^2$ edges and $\pi(F_n(a)) \le \frac{n^4}{2a}$.*

Proof: We build a spanner of G_n, $F_n(a)$, in the following way. We divide the grid into a^2 sectors. A point is in Sector (i,j) if its coordonnates in the grid (x,y) are such that $\frac{n}{a}i \le x < \frac{n}{a}(i+1)$ and $\frac{n}{a}j \le y < \frac{n}{a}(j+1)$. Each of these sectors has $(n/a)^2$ vertices. We call *center* of the sector (i,j) the vertex $((i+1/2)n/a, (j+1/2)n/a)$. We consider the $a \times a$ subgrid linking all the sectors' centers. We then connect all the remaining vertices of a sector to its center with a spanning tree. This way, we get $F_n(a)$. Figure 2 provide a sketch of the construction of the spanner.

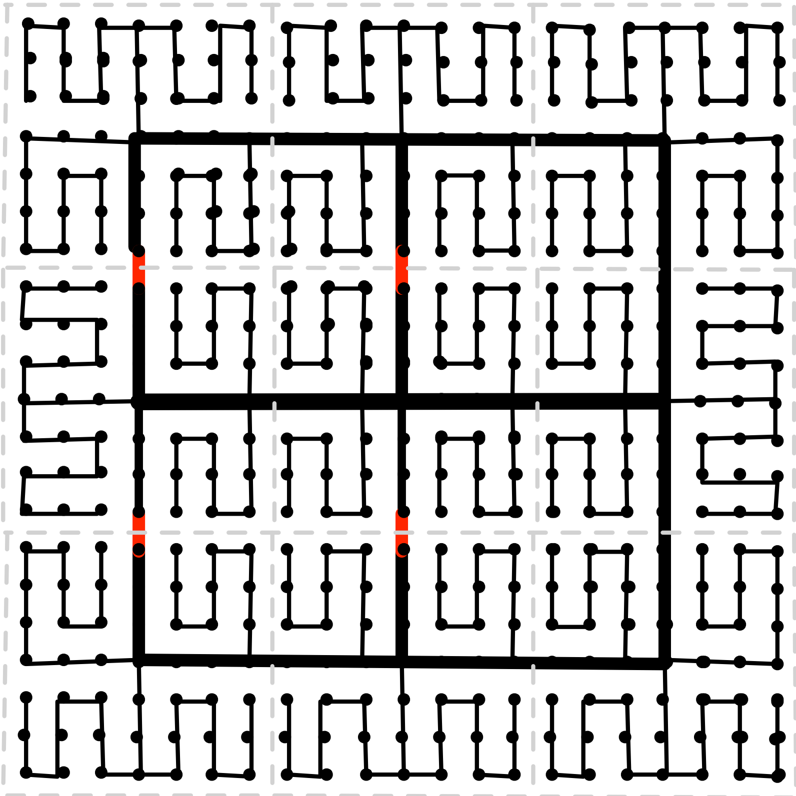

Figure 2: Spanner of Proposition 4 for $n = 21$ and $a = 3$. Edges of the $a \times a$ grid are in bold. Edges that are not in a spanning tree of G_n are in red. Sectors with $(n/a)^2 = 7^2$ vertices are separated by dashed gray lines.

We now build a routing R for $F_n(a)$. The demand between two vertices of the same sector are routed on the tree spanning their sector using the unique shortest path between them. The demand between two vertices of different sector is first routed to their centers, and then is routed in the $a \times a$ grid.

Let us compute the load of the routing R. We first consider the edges of the $a \times a$ subgrid. We know that an $a \times a$ grid has a routing with load $a^3/2$ (Proposition 1). Thus, we know that it also has a w-routing of load $wa^3/2$. Each vertex of the $a \times a$ grid receives the load of the $(n/a)^2$ vertices connected to it. Thus, we take $w = (n/a)^2$ and we obtain a w-routing of the $a \times a$ grid of load $\frac{a^3}{2}(\frac{n}{a})^4 = \frac{n^4}{2a}$.

We then consider an edge that does not belong to the $a \times a$ grid. The only paths that can use this edge are paths going from any vertex of the grid to a vertex of its sector. Thus, its load is smaller that $(n/a)^2 n^2 = \frac{n^3}{a^2}$. This load is smaller than the maximum load on the $a \times a$ grid as soon as $a^2 \geq 2a$ which means as soon as $a \geq 2$.

Therefore $\pi(F_n(a), R) = \frac{n^4}{2a}$.

Let us now consider the number of edges of the spanner $F_n(a)$. The number of edges necessary to connect all the nodes is $n^2 - 1$. If we choose well these edges, we just have to add a^2 edges to obtain the $a \times a$ grid (see Figure 2, additional edges are in red). $F_n(a)$ thus has $n^2 + a^2$ edges. We can improve the spanner by using the results of Section 2. In Theorem 1, we show that we can find a spanner of an $a \times a$ subgrid with $\frac{13}{9}a^2$ edges and a routing R' with the same load as a full grid with $2a^2$ edges. By doing so, we get a new spanner $F_n(a)$, with $n^2 + \frac{4}{9}a^2$ edges and $\pi(F_n, R') = \frac{n^4}{2a}$. □

We can rewrite the result of Proposition 4 to point out the impact of additional edges in general (Corollary 1) and when we start from a spanning tree (Corollary 2).

Corollary 1 *There exist:*

- *A spanner of G_n with $n^2 + p^2$ edges, and an asymptotic forwarding index of $\frac{n^4}{3p} \simeq 0.33\frac{n^4}{p}$;*
- *A spanner of G_n with $n^2 + p$ edges, and an asymptotic forwarding index of $\frac{n^4}{3\sqrt{p}} \simeq 0.33\frac{n^4}{\sqrt{p}}$.*

Proof: Direct by Proposition 4 setting $p^2 = \frac{4}{9}a^2$ or $p = \frac{4}{9}a^2$. □

Corollary 2 *There exists a spanner of forwarding index $\frac{1}{\alpha}\frac{3n^4}{8}$, that is a factor α less than the one of the optimum spanning tree, while using $\frac{64}{81}\alpha^2 \simeq 0.79\alpha^2$ additional edges compared to a spanning tree.*

Proof: Recall that an optimum spanning tree has forwarding index $\frac{3n^4}{8}$, see Proposition 2. Dividing it by α means getting the forwarding index $\frac{3n^4}{8\alpha} = \frac{n^4}{2(4\alpha/3)}$. This is achieved by the spanner $F_n(a)$, with $a = 4\alpha/3$. The spanner has an additional number of edges compared to the spanning tree equal to $\frac{4}{9}(4\alpha/3)^2 \simeq 0.79\alpha^2$. □

3.2 Lower bounds

Proposition 5 *There exist no spanners of G_n with $n^2 + p^2$ edges and a forwarding index less than $\frac{1}{9\sqrt{12}}\frac{n^4}{p} \simeq 0.032\frac{n^4}{p}$.*

Proof: Let us consider a spanner of G_n that has $n^2 + p^2$ edges. We build a multigraph in the following way. We start by assigning to every node a weight of 1. Then, while there is still a vertex with degree 1 or 2, we delete this vertex and the edges connecting it to the graph and divide its weight evenly among its neighbors; in case the removed vertex was of degree 2, we also connect the two neighbors afterwards. At the end of this process, we get a multigraph H such that the number of its vertices N' and the number of its edges M' are related by the following equation: $N' + p^2 = M'$. Indeed, every time a vertex is removed in the process leading to H, the number of edges is decreased by 1. Since all the vertices in H have a degree strictly greater than 2, we have $\frac{3}{2}N' \leq M'$. This implies with the previous equation that $\frac{3}{2}N' \leq N' + p^2$. Hence, we have $N' \leq 2p^2$. Notice that the total weight is equal to n^2. We now apply the weighted version of the planar separator theorem [6] on H: there exists a partition of the vertices of H into three subsets A, S, and B, such that each of A and B has at most a weight $2n^2/3$, S has less than $\sqrt{6}\sqrt{2p^2}$ vertices (The original graph is of a bounded degree 4.) and there are no edges with one endpoint in A and another endpoint in B. This directly gives an edge cut of the original graph which has less than $2\sqrt{6}\sqrt{2p^2}$ edges and which partitions the original graph's vertices into two subsets of size at most $2n^2/3$. Therefore, any routing of this spanner will induce, at least on one edge of the cut, a load that is greater than:

$$\frac{1}{3}n^2 \cdot \frac{2}{3}n^2 \cdot \frac{1}{2\sqrt{6}\sqrt{2p^2}} = \frac{1}{9\sqrt{12}}\frac{n^4}{p} \simeq 0.032\frac{n^4}{p}.$$

□

4 Conclusion

We succeeded at providing spanners of the $n \times n$ grid with a small number of edges for a given forwarding index. Such spanners are important for energy efficient networks in which the traffic has to be routed in the network while using a minimum number of equipments. The unused equipments are then turned off to save energy. We leave as open two problems.

We propose spanners with a number of edges of optimum order for a forwarding index. More precisely, we have provided spanners of the grid with $n^2 + \frac{4}{9}a^2$ edges and forwarding indices $\frac{1}{2a}n^4 (2 \leq a < n)$. We proved that it is impossible to have spanners with the same FI and fewer than $\simeq n^2 + \frac{4}{9}(0.1a)^2$ edges. It would be very nice to succeed in filling the gap between the lower bounds and the constructions.

Similarly, we describe spanners with $13/9n^2$ edges and with the same forwarding index of the full grid G_n. We proved that spanners with such a forwarding index should have at least $12/9n^2$ edges. Would it be possible to find spanners with such a number of edges?

Last, we focused on a specific network in this work, the square grid. We are also interested by more general graphs. In particular, the arguments to derive lower bounds can be used for more general planar graphs with bounded degrees. It would be very interesting to find results and constructions for other families of planar graphs.

References

[1] Bouabdallah, A. and D. Sotteau, *On the edge forwarding index problem for small graphs*, Networks **23** (1993), pp. 249–255.
URL `http://dx.doi.org/10.1002/net.3230230406`

[2] Chabarek, J., J. Sommers, P. Barford, C. Estan, D. Tsiang and S. Wright, *Power awareness in network design and routing*, in: *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, IEEE, 2008.

[3] Chiaraviglio, L., M. Mellia and F. Neri, *Energy-aware umts core network design*, in: *The 11th International Symposium on Wireless Personal Multimedia Communications*, 2008.

[4] Chiaraviglio, L., M. Mellia and F. Neri, *Reducing power consumption in backbone networks*, in: *IEEE International Conference on Communications*, 2009, pp. 1–6.

[5] de la Vega, W. F. and Y. Manoussakis, *The forwarding index of communication networks with given connectivity*, Discrete Appl. Math. **37-38** (1992), pp. 147–155.
URL `http://dx.doi.org/10.1016/0166-218X(92)90129-X`

[6] Djidjev, H. N., *On the problem of partitioning planar graphs*, SIAM Journal on Algebraic Discrete Methods **3** (1982), pp. 229–240.

[7] Fernandez de la Vega, W. and L. M. Gordones, *The forwarding indices of random graphs*, Random Structures and Algorithms **3** (1992), pp. 107–116.
URL `http://dx.doi.org/10.1002/rsa.3240030108`

[8] Giroire, F., D. Mazauric and J. Moulierac, "Energy Efficient Routing by Switching-Off Network Interfaces," IGI Global, 2012 pp. 207–236.

[9] Giroire, F., D. Mazauric, J. Moulierac and B. Onfroy, *Minimizing routing energy consumption: from theoretical to practical results*, in: *IEEE/ACM International Conference on Green Computing and Communications (GreenCom'10)*, Hangzhou, China, 2010.

[10] Giroire, F., J. Moulierac, T. Phan and F. Roudaut, *Minimization of network power consumption with redundancy elimination*, in: *IFIP Networking* (2012), pp. 247–258.

[11] Giroire, F., J. Moulierac and T. K. Phan, *Optimizing rule placement in software-defined networks for energy-aware routing*, in: *Proceedings of the IEEE Global Communications Conference (GLOBE-COM)*, Austin, United States, 2014.

[12] R, J. Baliga, R. Ayre, K. Hinton and W, *Energy consumption in IP networks*, in: *Optical Communication, 2008. ECOC 2008. 34th European Conference on*, 2008.

[13] Restrepo, J. C., C. G. Gruber and C. M. Machuca, *Energy profile aware routing*, in: *Communications Workshops, 2009. ICC Workshops 2009. IEEE International Conference on*, IEEE, 2009, pp. 1–5.

[14] Xu, J.-M. and M. Xu, *The forwarding indices of graphs – a survey*, CoRR **abs/1204.2604** (2012).

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